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An improved meshing method for shape optimization of aerodynamic profiles using genetic algorithms

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SUMMARY

One of the basic problems in fluid dynamics shape optimization is mesh generation. When analysis is performed using the finite element method, meshes of sufficient quality need to be constructed automatically. This work presents a structured meshing procedure that creates subdomains for generating good quality structured meshes in critical flow regions around aerodynamic profiles. Techniques of this nature enable other kinds of problems and geometries to be tackled. To demonstrate its capacity, it was applied to a straightforward shape optimization problem in fluid dynamics via genetic algorithms (GAs), including a preliminary efficiency study for different GA parameter combinations. Copyright © 2008 John Wiley & Sons, Ltd.

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KEY WORDS: shape optimization; genetic algorithm; structured mesh; computational fluid dynamics; finite element analysis; airfoil

1. INTRODUCTION

In recent decades advances in hardware have enabled the development of new optimization tools for design problems. The genetic algorithms (GAs) have proven their strength against local extrema and numerical noise in aerodynamic optimization and their validity in problems with constraints [1–6].

Meshing is a key issue in problems of shape optimization in search spaces where highly different geometries exist. A suitable meshing method for finite element computational fluid dynamics (CFD) analysis in GA optimizations must be both robust and accurate enough, because it is basic to be able to mesh every geometry in the search space, and also the mesh has to be well shaped in critical flow zones to provide accurate results.

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There are two ways to solve this issue: to mesh each geometry or to deform the previous mesh adapting it to next geometry. Mesh deforming methods [7–10] have proven their validity in gradient-based search where the optimization starts from a given geometry and no sudden changes in geometry take place. Other methods use mesh smoothing techniques such as Wang's [11], Laplace or Winslow to reduce mesh distortion, but they seem too expensive.

In this work a reliable meshing procedure for GA optimization problems is developed. Also a shape optimization system has been developed for fluid dynamic problems using a simple GA based on the David Levine PGA Pack library functions [12]. It has been applied to an optimization problem of bi-dimensional aerodynamic profiles with geometric constraints. The evaluation function calculates the flow using the Fidap CFD code based on finite elements. A study was carried out in order to obtain efficient combinations of the basic GA parameters.

This paper presents a method for creating meshes for profiles of highly different geometry with quality finite elements. To do this the mesh domain is divided into mesh blocks varying according to profile.

2. PROBLEM DEFINITION

An optimization problem of bi-dimensional aerodynamic profiles in turbulent incompressible flow is tackled.

For a successful optimization process, the search space must incorporate as much geometric flexibility as possible with as few design variables as possible. In this sense Bezier curves provide soft and flexible shapes when design variable intervals are appropriately defined.

Profile geometries are represented by two Bezier curves, one for the upper profile surface and another for the lower one, both of the form

$$x(t)=\sum_{i=0}^{n} c_n^i t^i (1-t)^{n-i} x_i, \quad y(t)=\sum_{i=0}^{n} c_n^i t^i (1-t)^{n-i} y_i \tag{1}$$

where n is the curve degree, $c_n^i = n!/i!(n-i)!$ and (x_i, y_i) are the $n+1$ curve control points, t being a parameter which varies at $[0,1]$.

A value of 4 was taken for n; hence, there are five bezier control points for each Bezier curve. First and last control points are fixed at $(0,0)$ and $(1,0)$, which are, respectively, the leading and trailing edges of the profile (see Figure 1). The abscissas of the three intermediate control points are fixed at 0, 0.3 and 0.7 m, respectively.

For this parametrization of geometry no constraint for the leading edge curvature exists.

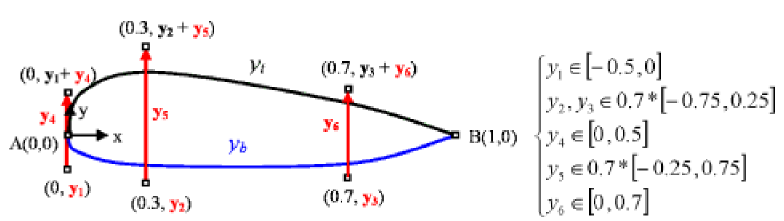

Figure 1. Design variables defining the profile and their definition intervals.

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3. MESHING METHOD

The robustness and accuracy of the meshing process is a mayor key player in aerodynamic shape optimization in search spaces with high geometric flexibility. In this sense GAs are capable of avoiding local extrema, but geometries vary drastically from one evaluation to the next.

A meshing method must have three characteristics to be suitable for optimization problems using GAs. First, it must be able to mesh every geometry in the search space. Second, as economy is a fundamental issue in GA optimization, it must allow mesh density to change rapidly within the domain so that there is only high-density mesh where it is needed (in critical flow zones). Third, in these zones the finite elements must have a good aspect ratio and not much distortion to guarantee that the flow is accurately calculated.

There are two kinds of meshing schemes: unstructured and structured ones. Unstructured meshing schemes allow fast variation of element size but they usually fail at meshing some geometries with the subsequent loss of information for the GA. On the other hand, algorithms for structured meshes can mesh every geometry but the shapes of the finite elements can be too distorted, this means unacceptable. This work proposes a mesh procedure based on defining subdomains in the computational domain, which are then used for multiblock-type mesh generation. The boundaries of the subdomains in critical flow zones are calculated individually according to the shape.

Computational domain is divided into mesh blocks as shown in Figure 2. Thus, the critical flow zones—wall proximities and wake—are enclosed within the upper mesh blocks $B1_t$, $B2_t$ and $B3_t$ and the lower $B1_b$, $B2_b$ and $B3_b$, where a fine mesh will be constructed. Critical subdomains $B2_t$ and $B2_b$ are defined by boundary curves C1, C2 and C3, which vary depending on the airfoil geometry to guide the mesh so that the elements inside have a good shape. $C1$, $C2$ and $C3$ are NURBS-type curves that interpolate points 1–9 (see Figure 2).

Point 1 allows minor element distortion on the leading edge. It is located at the intersection between horizontal $y=0.03d$ and bisector b_A of the angle forming the secant to the profile at $t=0.03$ and axis X (see Figure 3). Thus, the larger the curvature radius on the leading edge, the farther to the left this point is located to leave sufficient space for the mesh between this block boundary curve and profile.

Point 2 has ordinate d, and its abscissa is not too far from $x=0$.

For analogous reasons to the previous, point 9 is the intersection of horizontal $y=0.03d$ and bisector b_B of the angle formed by the tangent at B and abscissa axis. For point 8, the ordinate is d and its abscissa is close to $x=1$.

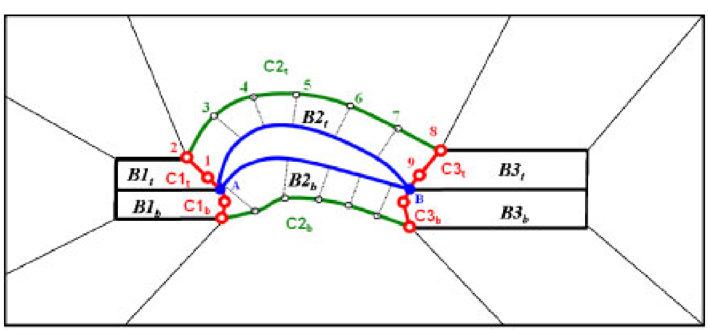

Figure 2. Domain structure divided into mesh blocks.

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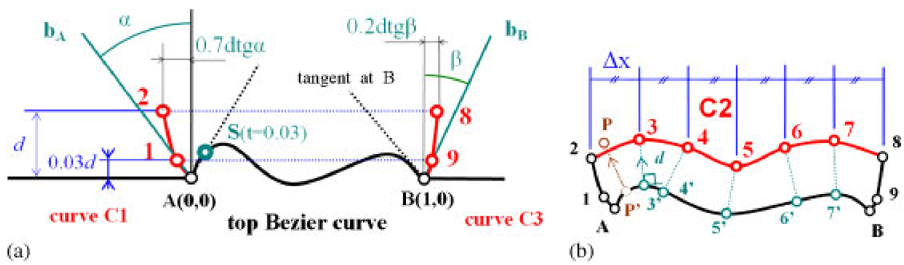

Figure 3. Building boundary curves (a) C1 and C3 and (b) C2.

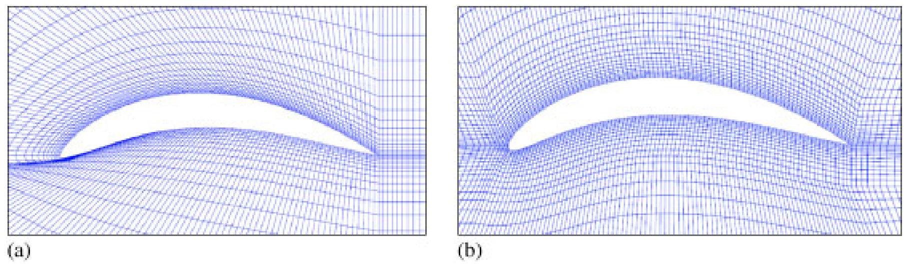

Figure 4. (a) Self-overlapping mesh for a simple geometry airfoil for an inappropriate domain structure and (b) mesh around same geometry generated with the developed meshing procedure.

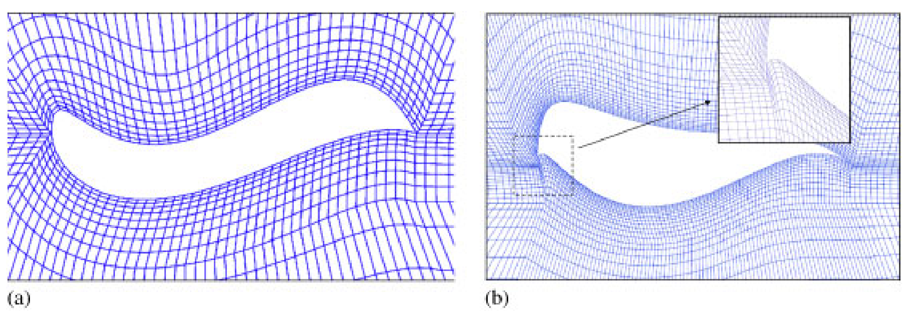

Figure 5. Two examples of meshes around profiles for (a) a coarse grid and (b) a fine one.

Curve $C2$ is designed to have a soft curvature. Value Δx equals $(x_8 - x_2)/6$, and it has been proven experimentally that taking the value $d = 0.12$, good results are obtained (see Figures 4(b) and 5(b)).

The five $C2$ points 3, 4, 5, 6 and 7 are projected from the profile a distance d, and they are sufficiently separated from each other as shown in Figure 3. To obtain these points, auxiliary point P is obtained projecting a point P' from the profile in the normal direction to the same. P' is travelled along the profile starting from $t = 0.01$ to increases of $\Delta t = 0.04$. If the horizontal distance between the projected point $P(t = 0.01)$ and point 2 exceeds Δx, that is if $x_p > x_2 + \Delta x$, then P is selected as point 3 of curve $C2$. Otherwise, t is increased until $P(t)$ exceeds the said distance.

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To calculate the four remaining points new $P(t)$s are calculated whose abscissas are compared with x_i on the last point i chosen, $i=3,\ldots,6$. The lower C_i boundary curves are traced in the same way. Once the boundaries of mesh blocks $B2t$ and $B2b$ are obtained, all the mesh blocks are meshed by using Fidap structured mesher.

In this way, several layers of good quality elements to a distance d from the profile are allowed to be constructed, as it can be seen in the examples in Figures 4(b), 5(a) and (b) for radically different geometries and mesh densities. The suitability of the proposed domain structure is also shown in Figure 4 for a simple geometry profile comparing generated meshes with this procedure and with another typical one.

Method validity was checked through numerous runs. Moreover, the method can be adapted to other problems and different domains.

4. PRELIMINARY EXPERIMENT

A preliminary experiment was performed to find efficient combinations of parameters for a basic GA. The aim is to optimize the shape of bi-dimensional aerodynamic profiles of 1 m of chord, for a flight speed of 240 km/h at zero angle of attack. The optimization criterion was chosen to maximize the ratio between lift and drag forces per unit span (L and D, respectively) with a constraint of minimum thickness ($h_{\min}$) of 12 cm.

In search spaces with constraints the optimum solution may be in the intersection between the constraint conditions and search space. The non-linear constraint is inserted in the objective function as a penalization as follows:

$$\Phi=\begin{cases} L/D & \text{if } h\geqslant h_{\min} \\ L/D-k(h-h_{\min}) & \text{if } h<h_{\min} \end{cases} \tag{2}$$

where k is a positive constant that reduces the fitness of too thin profiles. This function does not directly eliminate the forbidden region chromosomes but enables the selection of individuals close to the frontier, which present a high value of L/D. A value of $k=2$ allowed the selection of good individuals with a very little less thickness than the thickness limit, which permits to take benefit for the evolution of some valuable genetic information.

Fidap commercial code was used to solve Navier–Stokes' equations with k–ε turbulence model using a segregated method. To prevent sudden changes in the pressure field along iterative analysis, which might lead to divergence, a relaxation factor value of 0.5 was used, which made possible the convergence of the analysis of every evaluation.

Tournament selection genetic operator was used, and a generational replacement model was opted for. Tests were performed for different combinations of population size and crossover and mutation rates for several levels of each as shown in Table I. Ten runs were performed per combination. The termination criterion was established based on number of generations, in this case 100.

It was noted that the best GA solutions were obtained for populations of 800 chromosomes (see Figure 6), but similar aptitude values can be found for populations of 400 individuals with less cost. A mutation rate value of 0.01 showed best performance. Nevertheless, moderate values of crossover rate does not appear to significantly affect maximum fitness.

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Table I. Results for each combination of GA parameters. Ten runs were performed for each one.

Population size	200			400				800		
Mutation rate	0.001		0.01	0.001		0.01		0.001		0.01
Crossover rate	0.5	0.65	0.65	0.5	0.65	0.5	0.65	0.5	0.65	0.65
Best individual average	25.7	24.5	25.1	27.7	24.9	27.0	30.5	29.6	29.1	31.1
Best individual found	28.4	26.9	29.0	31.2	26.7	33.3	33.9	34.3	32.9	34.5

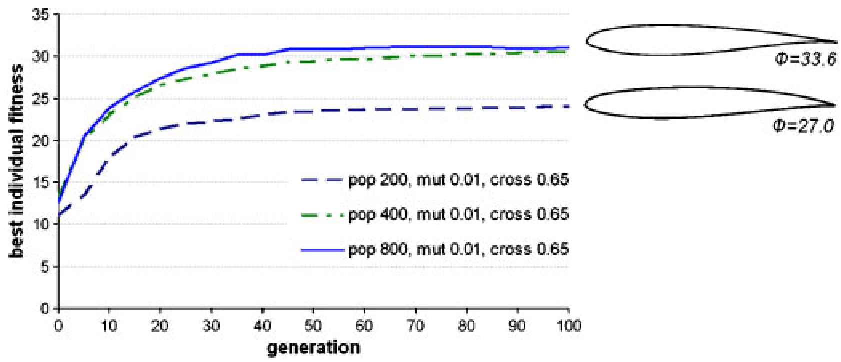

Figure 6. Best chromosome evolution for different population sizes.

Higher values of mutation rate did not present a remarkable reduction in fitness, which may be due to a lack of constructive blocks. To refute this hypothesis, a series of runs was performed via a random search of 80 000 individuals, i.e. the same number of evaluations as the largest GA runs. The best individual found had an aptitude value of 25, and the mean of each run best was 21, proving that the GA is more efficient than the random search.

5. CONCLUSIONS

An automatic meshing method suitable for shape optimization problems has been developed. It is capable of generating a quality structured mesh for any geometry in the search space. This meshing procedure allows shape optimization in search spaces with radically different geometries.

A shape optimization system has been developed for fluid dynamic problems, where an optimization algorithm (GA) was combined with a commercial finite element code to solve the said problems.

System robustness has been verified for the specific optimization problem of an aerodynamic profile. In addition, a preliminary experiment was performed to find efficient parameters of the optimization algorithm for this problem.

Application of this kind of technique in the future to multiobjective optimization of airfoils at different angles of attack has been considered. The validity of the k–ε turbulence models comparing their numerical results with wind tunnel experiments of known profiles will also be investigated.

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